**MF 796: Computational Methods of Mathematical Finance**

Professors Christopher Kelliher and Eugene Sorets Spring 2020

**Problem set # 1**

Due: Wednesday, January 29, by 8 am

1. **What areas of finance** are of most interest to you (e.g. big data, trading, portfolio management, risk management, derivative pricing, analyzing complex derivatives)?

The areas of finance of most interest to me are big data and risk management. First of all, the big data can offer many effective skills in data analysis, and these skills will have positive effects on the financial working in the future. Moreover, I think that risk management is significant during investment job. When I work in ZhongDe Securities, my manager always say this program will have significant effects on the financial working, and this program captivated me from the decision of investment strategies.

1. **What is the primary reason for your interest in this course?**

The primary reason is that this course includes that many computational problems in financial institutions, and these skills will help me to increase efficiency in financial working. On the other hands, this course will have a program and it will involve many the applications of programming technique, these factors will be an important treasure in the financial working in the future.

1. **List all programming languages you have used** and your level of familiarity with each.

I have used two programming languages include that R and Python, and I am better in Python than R.

1. **Option traders often say that when buying options we get gamma at the expense of theta.** What do you think they mean?

Option traders often say that when buying options we get gamma at the expense of theta, because they believe that Black-Scholes models can explain the process of buying options. By the Black-Scholes models, we can get

In here we can use gamma and theta to replace the partial derivatives, like

By the increasing in , will be larger. Obviously, the gamma will increase at the expense of theta. Therefore, option traders often say that when buying options we get gamma at the expense of theta.

1. **Consider the CEV Model:**

*dSt* = *rSt dt* + *σStβ dWt*

Assume a CEV model that is defined by the following parameters:

|  |  |
| --- | --- |
| *S*0 = 100 | (1) |
| *r* = 0*.*0 | (2) |
| *β* = 1*.*0 | (3) |
| *σ* = 0*.*25 | (4) |

* 1. Describe what each model parameter does.

*S*0 is initial price, and S is the spot price of underlying asset

*r* is the risk-free rate

*β* is positive constant

*σ* is a volatility parameter

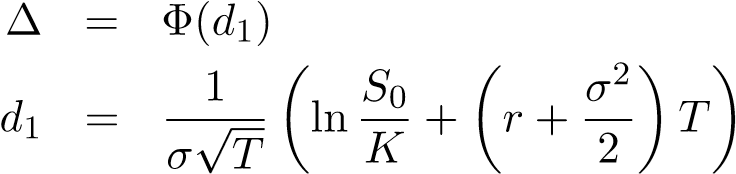
* 1. Price an at-the-money one year European call option via Monte Carlo simulation.(HINT: Increments of Brownian Motion are distributed normally with mean 0 and variance dt. One approach is to construct a set of paths by generating random normal variables to use as the increments of the Brownian Motion)

The code at the attachment.

* 1. Calculate the price of the same European call option via the Black-Scholes formula. Is this price the same as you obtained via simulation? Should it be? Why or why not?

By the Black-Scholes formula, the price of the same European call option is 9.95. The price is closed to me obtained via simulation. As the increasing in the number of trials, the value will converge to 9.95 by simulation, because the value of simulation will converge to the value of Black-Scholes formula.

* 1. In the Black-Scholes model, we know that the delta of a European call option is:

 *,*

Calculate the delta of an at-the-money European call option with one year to expiry.

The delta of an at-the-money European call option with one year to expiry approximately equal to 0.55.

* 1. Using the delta obtained above, calculate how many shares of stock you need to construct a delta neutral portfolio that is long one unit of the call option?

Construct a delta neutral portfolio, I need one unit of the call option.

* 1. Use simulation to estimate the payoff of the delta neutral portfolio obtained above. How does the payoff compare to the Black-Scholes model price of the option you obtained earlier? What conditions cause this portfolio to make money? Lose money?

The code at the attachment.

The Black-Scholes model price of the option is closed to the payoff.

When the option expires at the money or near the money, the portfolio will lose money. On the other hands, when the option expires deep in the money or out of money, the portfolio will make money.

* 1. Modify the model dynamics so that *β* = 0*.*5 and all other parameters are the same as in the original question. Using the same hedging portfolio perform another simulation to estimate the payoffs of the delta-neutral portfolio under these dynamics. Are the payoffs higher or lower? Why?

When the decreasing in *β*, the volatility of the stock will decrease. Furthermore, the variance of the stock price at maturity will be smaller. Therefore, payoffs will be lower, and the payoff approximately equal to 1.

* 1. Modify the model dynamics so that *σ* = 0*.*4 and all other parameters are the same as in the original question. Using the same hedging portfolio perform another simulation to estimate the payoffs of the delta-neutral portfolio under these dynamics. Explain the relationship between a delta-neutral portfolio and the *σ* parameter. ﻿

The payoff approximately equal to 15.4, because the increasing in *σ* will lead to the increasing in volatility, so the value will increase.

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